SHAPE DESCRIPTION
VIA WEIGHTED SKELETON PARTITION

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ABSTRACT

A shape description method is presented, which is based on the use of the weighted skeleton of a digital pattern. The skeleton is interpreted as a curve in the 3D space, where the three coordinates of any pixel are its planar coordinates and label. There, the skeleton is partitioned into rectilinear segments by means of a polygonal approximation. The partition allows one to decompose the pattern into simple parts. A merging step is also performed to reduce the number of regions of the final decomposition and to facilitate the description process. The process is fast because all the computations are performed on a limited amount of data, which are stored in vector form.

1. Introduction

When working with binary images, shape description is a key step for pattern recognition. If the pattern at hand has simple shape, e.g., it is a compact silhouette bounded by a convex contour, its description can be given in terms of global geometric features like area, perimeter and moments. These features are not enough to provide an adequate description of a pattern having complex shape, e.g., a pattern bounded by non-convex contour, which can be perceived as consisting of the union of more simple regions. In this case, shape description can be accomplished by following the structural approach: the pattern is decomposed into a number of simple regions that, by hypothesis, can be easily described by means of a suitable set of features; then, the description of the pattern is given in terms of the description of the obtained regions and of their interrelations.

The labelled skeleton of a single-valued digital pattern is a region-based representation system. It provides a convenient tool to analyse the shape of patterns perceived as union of elongated regions, being each skeleton branch representative of one of the constituting elongated regions. The skeleton is particularly suited to the structural approach to shape description, since it is naturally structured as a graph. In fact, each skeleton branch can be interpreted as a node (representing a pattern subset), while any crossing between branches identifies the interrelations of the corresponding regions and can be interpreted as an arc.

Different metrics can be adopted to drive the skeletonization process and to label the skeletal pixels. If the city-block or the chessboard distance functions are used, the obtained skeleton is rather sensitive to pattern orientation and its use for practical applications has to be limited to the case of patterns with fixed orientation. In fact, these distances provide a quite rough approximation of the Euclidean distance. Weighted distance functions [1, 2] allow one to obtain a better approximation, and to gain skeleton stability under pattern rotation.

In this paper we use a skeleton whose pixels are labelled according to the (3,4)-weighted distance function. The skeleton is interpreted as a curve in the 3D space, where the three coordinates of any pixel are the planar coordinates and the label. There, the skeleton is partitioned by means of a polygonal approximation. The obtained rectilinear segments are used to describe the corresponding regions. The description method has been inspired by previous papers [3, 4], relative to the case of ribbon-like patterns represented by their city-block distance labelled skeleton.

2. The Weighted Skeleton

Let $P=\{1\}$ and $\overline{P}=\{0\}$ be the two sets constituting a binary picture digitis ed on the square grid. The sets P and $\overline{P}$ are also referred to as the pattern and the complement. We assume that the 8-metric holds for P and the 4-metric for $\overline{P}$. Without losing generality, we assume that P is a connected set.

The (3,4)-weighted distance among two pixels p and q is the length of the shortest path (not necessarily unique) from p to q, where the two weights 3 and 4 are used to measure any horizontal/vertical unit move and any diagonal unit move, respectively.

The (3,4)-weighted skeleton S, [5], is the subset of P having the following properties: 1) it has the same number of 8-connected components as P, and each component of S has the same number of 4-connected holes as the corresponding component of P; 2) it is centred within P; 3) it is the unit-wide union of simple 8-arcs and 8-curves; 4) its pixels are labelled with their distances from $\overline{P}$; 5) it includes almost all the centres of the maximal discs of P.

Inclusion of almost all the centres of maximal discs guarantees a quasi faithful recovery of P, starting from S (the complete inclusion is not compatible with the simultaneous fulfilment of property 3) on skeleton thickness). Moreover, stability under rotation is achieved since the (3,4)-weighted distance provides a good approximation of the Euclidean distance. In fact, the disc associated by the (3,4)-weighted distance to any skeletal pixel is octagon-shaped, so that nearly the same number of discs fits the shape of the contour, whichever the orientation of the pattern.

We do not elaborate on how to compute the weighted skeleton, but simply outline the processing scheme, which includes three steps: i) computation of the (3,4)-weighted distance map DT, ii) identification of the set of the skeletal pixels (i.e., maximal centres, saddle pixels, linking pixels), and iii) reduction of the set of the skeletal pixels to unit width.

Detection of the centres of the maximal discs on the DT is a straightforward task, since the label of any pixel is related to the radius of the associated disc; detection of the saddle pixels requires the analysis of the neighbourhood of any pixel, so as to count the number of components of neighbours with smaller label and with larger label. Detection of the linking pixels is done by growing paths along the direction of the steepest gradient in the DT, starting from any already found skeletal pixel. The set of the skeletal pixels is reduced to unit width, by employing topology-and-end-point preserving removal operations.

The pixels of S can be classified into end points, normal points and branch points, by taking into account the number of components of neighbours not belonging to the skeleton and the number of neighbours belonging to the skeleton. An end point is a pixel of S having a unique (4-connected) component of neighbours not in the skeleton. A branch point is any pixel of S which is not an end point and has more than two neighbours in S. A normal point is a pixel of S which is neither an end point nor a branch point.

A skeleton branch is an arc of S entirely consisting of normal points, except for the extremes of the arc, which are end points or branch points. If neither end points nor branch points exist in S (i.e., the skeleton is a simple curve), any of its pixels is taken to represent both the extremes of the unique skeleton branch.

Each skeleton branch can be understood as the spine of a subset of the pattern. This is a simple region, if it satisfies the following two properties: 1) the contour arcs, which are common also to the contour of the pattern, are straight line segments; 2) its local thickness changes monotonically and linearly along the spine. The properties characterising a simple region are reflected by corresponding properties of the associated spine: this is a discrete straight line segment along which labels monotonically and linearly change.

In the following, unless differently specified, the labels of the skeleton pixels have to be understood as normalised labels, obtained by replacing any distance label p with the integer number k such that: $3(k-1) < p \leq 3k$

3. The Algorithm

A preliminary partition of the skeleton into its constituting branches is accomplished. To this purpose, each branch is traced from one extreme to the other extreme and the planar coordinates and label of each pixel are orderly stored in vector form. This partition of S is equivalent to a decomposition of P into elongated regions, i.e., the regions that could be obtained by individually applying to each skeleton branch the reverse distance transformation.

Each skeleton branch, interpreted as a curve in the 3D space (x, y, label), is then partitioned into a number of rectilinear 3D segments. To this aim, a polygonal approximation is performed. A split type algorithm, [6], is used so that the obtained set of vertices is not remarkably influenced by the order in which the pixels of the skeleton branch are examined. The complexity of this task is in order of n.log(n), where n is the number of pixels in the skeleton branch. Starting from the extremes of the branch (say A and B), new vertices are identified in a recursive way. The pixel C of the skeleton branch, whose Euclidean distance from the straight line (AB) is the maximal one, is taken as a new vertex, provided that the Euclidean distance is larger than an a priori fixed threshold θ. Vertex selection is then accomplished on the sub-branches AC and CB. The recursive process terminates when, for every pixel of each sub-branch, the distance from the corresponding straight line segment is not larger than the threshold θ.

The value for the threshold θ depends on the tolerance regarded as acceptable in the problem domain. In the experiments we have carried on, the value $\theta=1.5$ has been used to favour a rather faithful description of the pattern. Note that by assigning different values to θ, different descriptions of the pattern would be available. The descriptions are rougher and rougher as the threshold value increases.

Using the normalised labels in place of the distance labels allows us to treat uniformly both the planar and the label coordinates, since a displacement of at most one unit in each of the three directions is accomplished when passing from a skeletal pixel to one of its neighbours. In this way, the skeleton is a connected union of arcs and curves also in the 3D representation.

The skeleton segments obtained so far enjoy the properties characterising the spines of simple regions, and are used to extract geometric features of the represented regions. To this purpose, it is not necessary to reconstruct the regions by resorting to the reverse distance transformation. In fact, starting from the coordinates (x_1, y_1, $label_1$) and (x_2, y_2, $label_2$) of the vertices delimiting each partition component, an approximated version of the corresponding region can be built. This is obtained by drawing the discs associated with the vertices, and linking them by means of a trapezium-shaped strip having the partition component as its symmetry axis (see Figure 1). The tolerance used to perform the skeleton partition conditions the degree of approximation of the recovered regions.

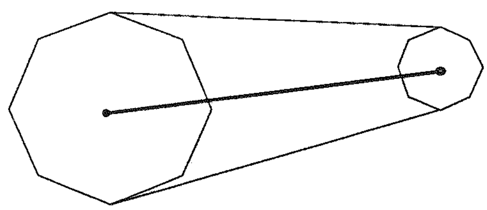
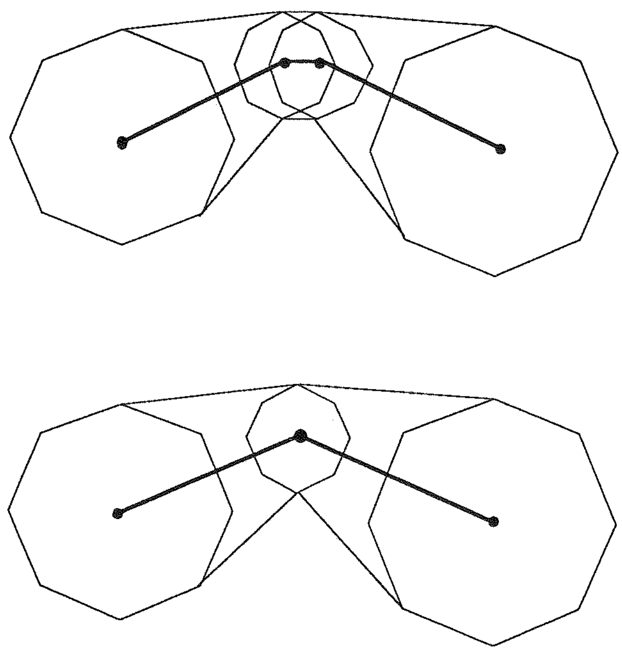


Figure 1. The region corresponding to a skeleton partition component.

Some of the regions derived from the partition of a skeleton branch are (almost completely) overlapped by adjacent regions. To facilitate pattern description, the corresponding spines should be identified as superfluous and removed. A spine is identified as superfluous if its length is smaller than the sum of the labels of the two vertices. When this is the case, and both the extremes of the spine are normal points, the segment is annihilated by moving its two vertices, which are also vertices common to contiguous segments, towards the middle of the superfluous spine. The label to be ascribed to the new vertex is computed by taking into account the relative distance and the labels of the two original vertices. An example is shown in Figure 2.

When an extreme of a superfluous spine is a branch point, its presence in the skeleton is necessary only to keep track of the interrelations among skeleton branches. Any such a spine simply plays the role of a linking element, but has no region representation power in the decomposition.

Although the regions remaining after superfluous spine annihilation are all significant, a merging process is accomplished to reduce the number of elementary regions which constitute the primitives for pattern description. In fact, the number of simple regions obtained so far could be quite large, due to the small value selected for the threshold during the polygonal approximation. We point out that the merging step could not be avoided, by increasing the value of the threshold used to perform the partition of the skeleton. In fact, representing with a unique spine (rectilinear in the 3D space) a region characterised by appreciable changes of global curvature and/or width could be too schematic, or even incorrect.



a)

b)

Figure 2. The initial partition of the skeleton into three components originates a decomposition, where the intermediate region is not significant, a). After the intermediate spine is annihilated, a more significant decomposition is obtained, b).

Purpose of the merging step is to consider the union of consecutive regions as a unique primitive, provided that only smooth changes (in orientation and/or thickness) occur when passing from one region to the successive one. The region obtained by making the union of simple regions is not a simple region, but its description can still be easily derived from the coordinates of the vertices delimiting the merged spines. The spine of the region union of simple regions is the concatenation of the merged spines.

Decision on region merging is taken by resorting to the 3D representation of the corresponding spines: two consecutive regions are merged if the distance of their common vertex from the straight line joining the remaining two vertices is less than an a priori fixed threshold.

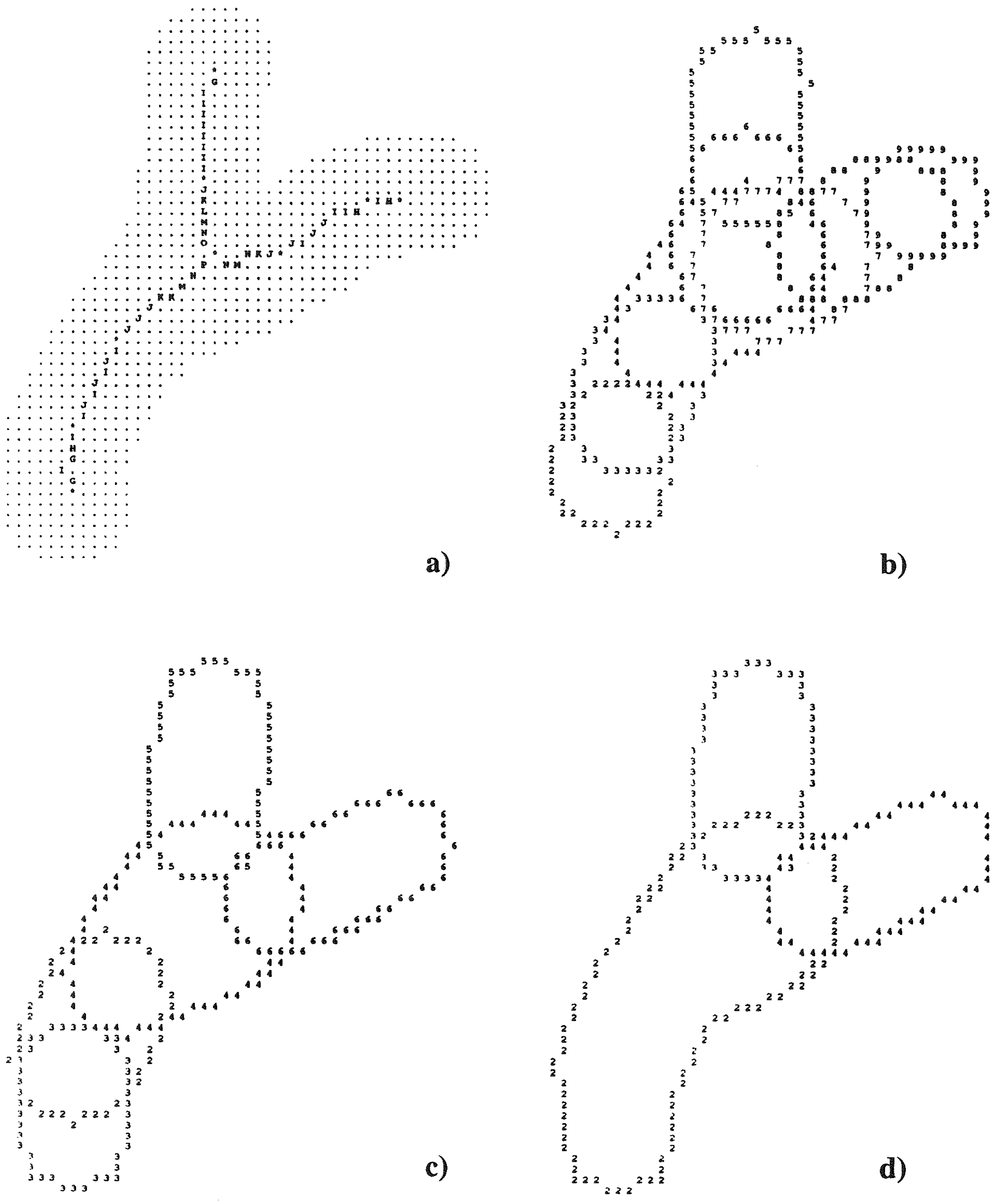


Figure 3. The initial partition of the (3,4)-weighted skeleton, where the vertices are denoted by stars a). The corresponding pattern decomposition, b). Intermediate result, after the superfluous spine annihilation, c). Final result, obtained after the merging step, d).

An example of the performance of the proposed algorithm is shown in Figure 3.

4. Conclusion

In this paper we have introduced a method for describing the shape of a digital pattern, represented by its weighted skeleton. A preliminary partition of the skeleton into the constituting branches is accomplished. Then, each branch of the skeleton is interpreted as a curve in the 3D space, where is partitioned into rectilinear segments by means of a polygonal approximation. This partition simulates the decomposition of the pattern into simple regions. An annihilation process is then performed to get rid of superfluous spines, representing regions almost completely overlapped by neighbouring regions. A merging step is then accomplished to reduce the number of elementary regions into which the pattern is decomposed. Besides avoiding some redundancy, the merging step allows us to obtain a final decomposition which is not strongly conditioned by the preliminary skeleton partition and results to be more in accordance with human intuition. The description of each region of the final decomposition can be achieved by using the coordinates and labels of the vertices of the involved skeleton segments, as well as a few notions of elementary plane geometry. The description of the pattern is obtained in terms of the description of its constituting regions and of their interrelations.

The process is very fast because all the computations are performed on a limited amount of data (the skeletal pixels and, afterwards, the vertices of the polygonal approximation), which are stored in vector form. The use of the (3,4)-weighted distance function to label the skeletal pixels allows us to have a skeleton stable under pattern rotation and, accordingly, the same description is expected for the pattern, whichever its orientation. Investigation in this respect is currently under development.

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