SKELETON SIMPLIFICATION THROUGH NON SIGNIFICANT BRANCH REMOVAL

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Abstract. Discrete and continuous skeletons suffer from the presence of non significant branches. This makes the skeleton structure complex and prevents to easily establish a correspondence between skeleton subsets and figure regions. We illustrate non significant branch removal techniques, which allow us to simplify the skeleton without reducing its representative power. To this purpose, a quantitative evaluation of the relevance of the region mapped in a skeleton branch is used to decide on branch removal.

Key words. Discrete skeleton, continuous skeleton, pruning

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1 Introduction

The skeleton is a convenient representation of figures which can be interpreted as constituted by the superposition of elongated regions. In fact, any skeleton branch is a unit wide set, centred within an elongated region and oriented along the directions of the main symmetry axes of the region. Beside symmetry, it also accounts for other shape properties of the region, such as elongation and width. The length of a skeleton branch can be used to evaluate the elongation of the represented region. In turn, quantitative information on region's local thickness can be achieved provided that the skeleton elements are labelled with their distance from the figure boundary.

Research on skeletonization has been influenced, at least implicitly, by the work of Blum on the continuous plane [1] dealing with the primitive notions of a symmetric point and a growth process. A symmetric point has equal distance from at least two different boundary points, and the growth process associates to each symmetric point the largest disc, centred on the point and fitting the shape. The skeleton is the collection of all the symmetric points and of the radii of the associated maximal discs. A conspicuous number of papers dealing with skeletonization can be found in the literature. Most of them refer to the computation of discrete skeletons ([2-7]), mainly because they appear as a natural choice when working with digital figures. On the other hand, a set of vertices approximating figure boundary in the continuous plane may also be available. In this event, the vertices can be used to compute the Voronoi graph and, hence, the continuous skeleton, without performing any shape digitisation. For this reason, and due to the no longer prohibitive computation cost of the Voronoi graph, the interest towards continuous skeletonization has considerably increased in the last decade([8-10]).

A problem common to discrete and continuous skeletonization is the creation of non significant (peripheral) skeleton branches in correspondence with regions whose perceptual relevance is disregardable. This makes the skeleton structure complex and severely conditions the possibility to use the skeleton for shape analysis. A less complex skeleton can be obtained by cleaning the input figure prior to skeletonization. However, cleaning produces partially satisfactory results, and limitedly to the case of non significant branches originated due to noisy contour configurations.

The design of suitable pruning techniques is a convenient way to simplify the structure of the (discrete or of the continuous) skeleton, in such a way that a correspondence can be established between the skeleton branches remaining after pruning and significant figure regions. Since the pruned skeleton represents the smoothed version of the figure, resulting after flattening some of its protrusions, pruning criteria can be based on the geometric properties which allow discrimination between the protrusions significant in the problem domain and those that can be flattened.

This paper provides different criteria for pruning the (discrete and the continuous) skeleton, by totally removing or partially shortening unwanted peripheral branches, without significantly altering the representative power of the skeleton. Sections 2 and 3 briefly introduce discrete and continuous skeletonization; the proposed criteria for branch removal are illustrated in Section 4 and some concluding remarks are given in Section 5.

2 Discrete skeletonization

Discrete skeletonization can be achieved by repeatedly applying a contour peeling process (e.g. [2]), or by using a distance map based approach (e.g., [7]). The latter method is more directly related to the Blum's notions of a symmetric point and a growth process. In fact, in the distance map the pixels are labelled with their distance from the complement of the figure, computed according to a given distance function [11-13]. Thus, the pixels symmetrically placed within a digital figure, as well as their associated radii, can be easily found in the distance map.

Different distance maps originate different skeletons for the same figure. City-block and chessboard distances have been widely used in the past, as a natural choice on the discrete square grid. However, they provide a rough approximation to the Euclidean distance and originate skeletons whose structure is strongly conditioned by figure orientation. The (3,4)-weighted distance and the (5,7,11)-weighted distance, introduced in [12], provide a better approximation to the Euclidean distance, and accordingly allow to originate skeletons almost stable under figure rotation.

Every pixel in the distance map can be interpreted as the centre of a disc fitting the figure, and having radius equal to the label of the pixel. The disc is a polygon approximating the Euclidean circle to a different extent, depending on the adopted distance function. Discs obtained by the city-block distance and chessboard distance are 4-side polygons, while those obtained via the (3,4)-weighted distance and the (5,7,11)-distance are 8-side and 16-side polygons, respectively.

If the centres of the maximal discs (i.e., discs that are not included by any other single disc) are ascribed to the skeleton, skeletonization becomes reversible, since the union of the maximal discs coincides with the figure. Detection of the centres of the maximal discs in the distance map can be done by suitably comparing the label of any pixel (i.e., the radius of the associated disc) with the labels of its neighbours (i.e., the radii of the associated discs). Generally, the set of the centres of the maximal discs is not connected, even for a connected figure, and is more than one pixel wide, wherever the thickness of the figure is given by an even number of pixels. To gain skeleton connectedness, further skeletal pixels (the saddle pixels, and the linking pixels) have to be found on the distance map. Detection of the saddle pixels can be done by analysing the neighbourhood of any pixel, so as to count the number of components of neighbours with smaller label and with larger label. Detection of the linking pixels can be done by growing paths along the direction of the steepest gradient in the distance map, starting from any already found centre of maximal disc or saddle pixel. Finally, the set of the skeletal pixels can be reduced to the unit wide skeleton, by employing topology preserving removal operations, designed in such a way to prevent excessive shortening of the skeleton branches.

The figure can be almost completely recovered by applying to its skeleton the reverse distance transformation. Complete recovery is not compatible with the requirement that the skeleton be one pixel wide. In fact, this requirement forces removal of a number of centres of maximal discs from the set of the skeletal pixels. Maximal discs, associated with skeleton pixels sufficiently close to each other, partially overlap so that the set of the maximal discs provides a covering of the figure, which is not a partition. The contour of a maximal disc and the contour of the figure share one, two or more connected subsets, each of which may include more than one pixel.

3 Continuous skeletonization

Differently from the discrete case, continuous skeletonization does not require bit map image digitisation. The obtained skeleton is a graph, which is computed starting from a polygonal approximation of the continuous shape (provided, for instance, by segmentation methods using a deformable curve model). The vertices of the polygonal approximation in the continuous plane sample the boundary of the continuous shape, and are called the sampling points. A measure of the quality of the approximation is given by bounding the greatest distance between two neighbouring sampling points on the boundary. The more numerous those sampling points, the more accurate the approximation.

Continuous approaches are based on the computation of the Voronoi graph [14]. For a finite set of seeds E, the Voronoi graph consists of the boundaries of the Voronoi regions. The Voronoi region of a seed is the set of points of the plane closer to this seed than to any other seed. The Voronoi regions are polygons and the Voronoi graph is made up of vertices and straight-line segments (see Figure 1a). The dual of the Voronoi graph is the Delaunay triangulation. It consists of triangles whose circumscribed circles do not contain any seed (see Figure 1b). After the Voronoi graph of the sampling points has been computed, a subgraph can be extracted to approximate the skeleton of the continuous shape. The differences among the skeletonization methods in the recent literature regard the selection of the best subgraph to approximate the skeleton. A possible choice consists in selecting the set of Voronoi vertices that are inside the shape (see Figure 2). Indeed, for sufficiently regular shapes, it has been proved that the Voronoi vertices of the sampling points tend

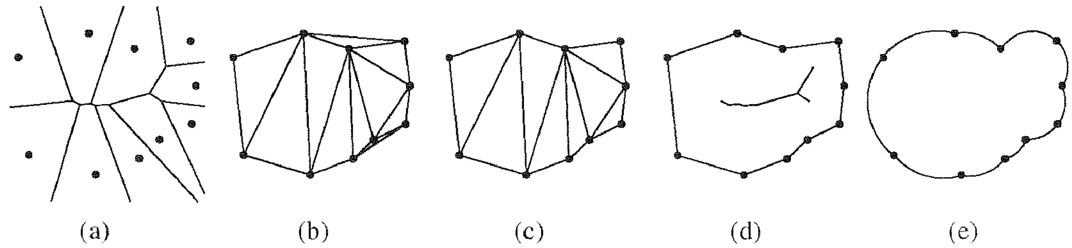

Figure 1: (a) Voronoi graph; (b) Delaunay triangulation; (c) Partition of the shape with Delaunay triangles; (d) Approximate skeleton; (e) Reconstruction of the shape using the Delaunay disc.

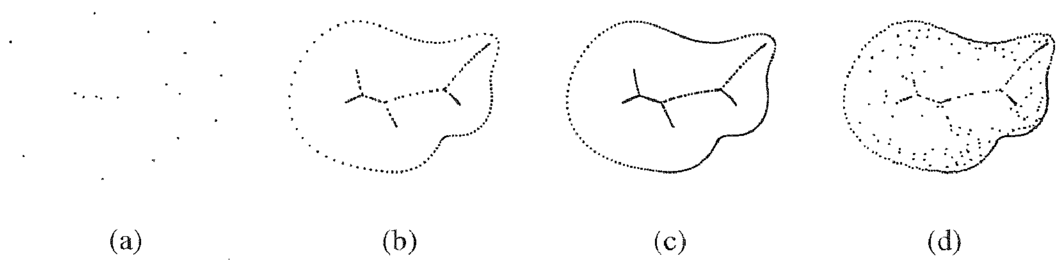

Figure 2: The set of Voronoi vertices included in the shape tends to the skeleton as the sampling density increases. From (a) to (c), the shape is sampled by 10 points, 50 points and 200 points; In (d), some noise is added to the 200 sampling points. The location of the Voronoi vertices is consequently modified.

to the complete skeleton (i.e., the endoskeleton and the exoskeleton of the shape) when the sampling points tend to the shape boundary [15]. However, this choice leads to a set of disconnected points and consequently, contains no information on the topology of the original shape. To overcome this problem, a connected overset of the inside Voronoi vertices can be taken [8,9,16,17]. The computation time of continuous methods comes down to the computation time of the Voronoi graph which is $O(nlogn)$, where n is the number of sampling points.

The method proposed in [17] is particularly interesting because it ensures homotopy between the approximate shape and the approximate skeleton. Assume the shape to be partitioned with Delaunay triangles (Figure 1c). Then, the skeleton (Figure 1d) consists of the Voronoi vertices associated with Delaunay triangles contained inside the shape, and of the straight-line segments connecting the vertices. Two vertices are connected by a segment if their associated triangles are adjacent. In order to reconstruct the continuous shape, one can use either the Delaunay triangles associated with the Voronoi vertices, or the Delaunay discs (Figure 1e). A Delaunay disc is the disc circumscribed to a Delaunay triangle. Triangles provide a partition of the region of the plane enclosed by the initial polygonal approximation, while the discs partially overlap. Their union tends to the continuous shape, when the density of the sampling points sufficiently increases.

4 Branch removal

4.1 Preliminary notions

Unless explicitly pointed out, from now on the term skeleton will be used to equivalently refer to the discrete and the continuous skeleton, and the term element will denote a skeletal pixel and a Voronoi vertex, respectively.

The elements of the skeleton can be classified as end points, normal points and branch points, depending on the number of neighbouring elements they have in the skeleton. End points have only one neighbouring element, and normal points have exactly two neighbouring elements in the skeleton. Branch points have exactly three (more than two) neighbouring elements in the continuous (discrete) skeleton. A skeleton branch is a subset of the skeleton entirely consisting of normal points except for two elements, called the extremes of the skeleton branch, that are end points or branch points. When one extreme of a skeleton branch is an end point, the skeleton branch is termed a peripheral skeleton branch. A correspondence can be established between skeleton branches and figure regions. In particular, peripheral branches can be associated with figure protrusions.

To avoid topology modifications of the skeleton, only peripheral branches can be pruned. Removing (partially or totally) a skeleton branch is equivalent to flattening the corresponding figure protrusion. In our opinion, only tapering protrusions (i.e., protrusions whose local thickness decreases when proceeding towards the periphery of the protrusion) should be flattened, but not bulbous protrusions (i.e., protrusions that are linked to the figure by a neck), because the latter might be regarded as significant regions in their own right. Thus a skeleton branch should be pruned only if the sequence of the radii of the discs associated to the elements encountered along the branch, starting from its tip, never decreases.

Pruning may involve either partial shortening or complete deletion of a peripheral branch. Starting from the end point, the elements of the branch are checked one after the other against a given pruning condition. Pruning can be done as far as the pruning condition is satisfied. In the following, we denote by p the end point of a peripheral skeleton branch, and by q any successive more internal element along the same branch. If the pruning condition is satisfied up to the element q, then all the elements of the branch from p to q, q excluded, are removed.

Generally, the pruning condition should prevent excessive shortening of skeleton branches, as this may result in loss of skeleton representation power. Thus, pruning should be based on a measure of protrusion relevance and the only branches to be pruned are those associated with protrusions regarded as non meaningful according to the relevance measure.

If all the branches sharing a branch point are totally deleted, new peripheral branches are originated in the modified skeleton which can be furthermore subjected to pruning, provided that the relevance of the regions corresponding to the already pruned branches is recorded. In this way, decision on pruning can be taken by using the relevance of the complex region mapped in the union of the current peripheral skeleton branch with the neighbouring, already pruned, skeleton branches.

The pruning criteria we discuss in the following can be applied to discrete or continuous skeletons, obtained by using any algorithm available in the literature. The examples shown in this paper refer to the discrete skeleton [7], computed by using the (5,7,11)-weighted distance, and to the continuous skeleton [10]. These skeletons are shown superimposed over a test pattern in Figure 3a,b, as they result before performing any pruning.

4.2 Branch removal criteria

Branch length. The length of a skeleton branch, i.e., the number of elements constituting the branch, can be used to decide on branch removal. A peripheral branch can be entirely removed if its length is below an a priori fixed threshold. This criterion has not general applicability, as the length of a branch depends also on the thickness of the region from which the corresponding protrusion sticks out. As an example, see Figure 4, where a very small protrusion on the bottom side of the pattern is associated with a skeleton branch whose length is almost equal to the length

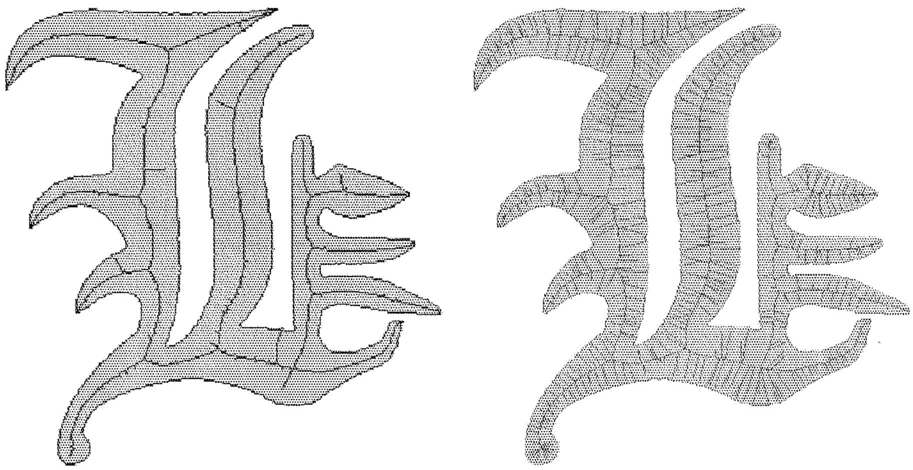

Figure 3: Discrete skeleton, (a), and continuous skeleton, (b), before pruning.

of the remaining two branches. In general, the length based pruning criterion can be safely used only to remove very short branches, say consisting of 1 or 2 elements, as these are almost surely noisy branches. It can also be used for particular classes of figures, as those constituted by the superposition of elongated components having constant thickness (e.g., chromosomes).

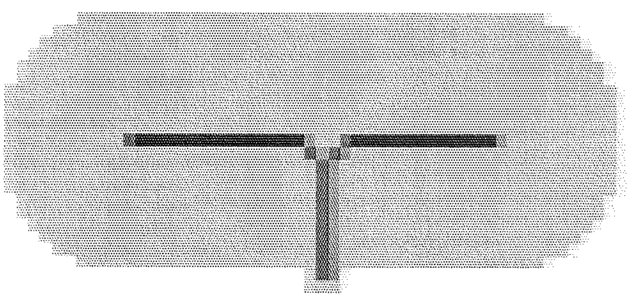

Figure 4: A noisy branch originating from a small protrusion on the bottom side of the pattern has length almost equal to that of the remaining significant branches.

The length based criterion can be improved to increase its applicability. In the discrete case, the number (and the distribution) of the centres of the maximal discs along a branch, rather than the number of pixels constituting the branch itself, can be used to decide on pruning. In fact, significant branches include a remarkably larger number of centres of maximal discs, than that present on noisy branches having the same length.

By using the Euler constant, it can be proved that the number of vertices of the continuous skeleton is related to the number of sampling points along figure boundary. If the sampling points are regularly spaced, then, computing the number of vertices of a skeleton branch is equivalent to computing the length of the boundary of the protrusion associated with that branch. Pruning methods based on the computation of the length of the boundary have been proposed in [9].

Protrusion elongation. Intuitively, a (portion of a) branch can be safely pruned if the difference in elongation between the two regions, respectively corresponding to the entire skeleton branch and to the pruned skeleton branch, is negligible.

Let r and R be the radii of the discs associated with p and q, and let d be the distance between p and q. The quantity $(r - R + d)$ measures the distance between the contours of the regions, respectively associated with the entire branch and with the branch pruned up to q. This value can compared with a threshold ϑ, to be fixed depending on the accepted tolerance in figure recovery. Pruning can be done up to the most internal element q such that $(r - R + d) \leq \vartheta$.

The above criterion is strictly based on the difference in elongation and does not take into account protrusion sharpness. It can be suitably modified, so as to prevent pruning of skeleton branches corresponding to sharp protrusions. Indeed, protrusion sharpness depends on the difference in radii as well as on the distance between the two extremes of the corresponding skeleton branch, i.e., it depends on the tangent of the angle β, as shown in Figure 5. To take into account both elongation and sharpness, pruning should be accomplished only provided that it is: $(r - R + d) \leq \vartheta \times (R - r + 1)/d$.

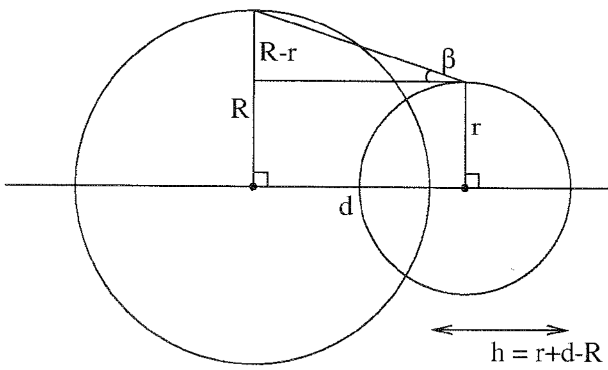

Figure 5: The angle β changes with protrusion sharpness.

The performance of pruning based on elongation and sharpness is shown in Figure 6, referring to the discrete skeleton. The value used for the threshold was $\vartheta = 2$, which has been found to be adequate for removal of noisy branches. White regions inside the shape contour identify the pixels non recovered by the pruned skeleton.

For completeness, we point out that if a large value is assigned to ϑ, so as to produce a more important figure smoothing, the geodesic distance between p and q should be employed in place of the distance d, when evaluating $(r - R + d)$. Otherwise, the pruning condition might be satisfied also by two elements p and q of a significant skeleton branch, along which relevant curvature changes occur.

Protrusion area. The difference in area between the two regions, respectively corresponding to the entire skeleton branch and the skeleton branch that would result after pruning, can be used to decide on branch removal.

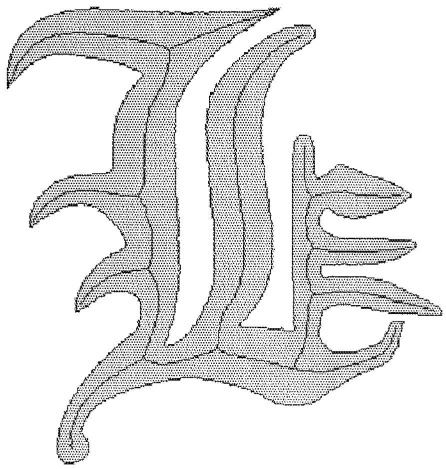

Figure 6: Effect of elongation-based pruning on the discrete skeleton.

The difference between the region corresponding to the skeleton branch including all the elements from p to q, and the region associated to the element q alone, defines the protrusion that would be flattened by pruning the skeleton branch up to q. The area P of the protrusion can be directly compared with a threshold or, preferably, it can be compared with the area F of the whole figure (easily available in both the discrete and the continuous case) or with the area D of the region associated to the element q. Comparing P with F allows one to use the same threshold whichever is the size of the figure at hand. Pruning will be equally effective on equally sized protrusions. Comparing P with D makes pruning more context dependent.

In the continuous skeleton case, protrusion area evaluation can be accomplished easily, since the skeleton vertices are associated with non overlapping triangles. In turn, the Delaunay discs partially overlap each other and also one Delaunay disc can overlap partially several Delaunay triangles. One can make use of this remark to provide a better pruning criterion as well as a more faithful shape reconstruction. The key idea

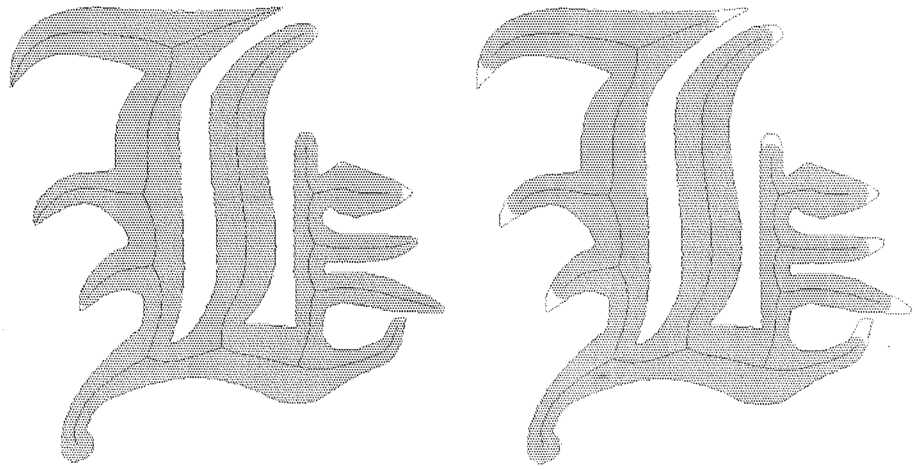

Figure 7: Effect of the area-based pruning on the continuous skeleton: when the area of the protrusion is compared with the area of a single disc, (a), and with the area of the entire shape, (b).

is that of using the Delaunay triangles while evaluating the contributions provided by the Voronoi vertices that are going to be removed by pruning (i.e., the vertices from p to q, q excluded), and to use the Delaunay discs when evaluating the area of the region associated to q and, in general, for shape recovery. If the current protrusion mapped in the skeleton branch from p to q is significantly overlapped by the Delaunay disc associated to q, then the branch can be safely removed; a rather faithful recovery is still possible, provided that the shape is reconstructed by using the union of the Delaunay discs rather than the union of the triangles. The area P of the protrusion is computed by adding the area of the Delaunay triangles associated with the skeleton vertices from p to q, q excluded, while D is the area of the Delaunay disc associated to q. The branch is shortened up to q if it results: $P \leq \vartheta \times D$. Alternatively, if F denotes the area of the shape, pruning can be accomplished provided that $P \leq \vartheta \times F$. In both cases, the value of ϑ depends on the tolerance in figure recovery. In particular, when comparing P with D, the value of ϑ ranges between 0 (which prevents any simplification) and 1 (which removes every branch).

Figure 7 shows the continuous skeleton after applying the area-based pruning; shape reconstruction is done by employing the Delaunay discs. Both the polygonal approximation of the initial shape and the reconstructed shape (grey region) are illustrated. In Figure 7a , the area P of the protrusion is compared with D and it is $\vartheta = 0,5$; in Figure 7b, P is compared with the area of the entire shape and it is $\vartheta = 0,005$.

The computation of the area of the protrusion is not straightforward in the discrete case, since the maximal discs partially overlap. Only for skeletons driven by the city-block and the chessboard distances (i.e., in case of square-shaped discs), convenient algorithms have been introduced to compute the area of the union of the maximal discs [18]. For the general case of skeletons driven by weighted distances, providing more rounded discs, an approximated evaluation of the protrusion area can be computed as $(R^2 - r^2) \times (tan\alpha - \alpha)$, where α is the angle shown in Figure 8.

Analogously to the continuous case, the area of the protrusion can be compared with the area F of the whole figure, or the area D of the disc associated with the element q. Figure 9, shows the effect of the area-based criterion on the discrete skeleton, when the area of the protrusion is compared with F and the threshold is $\vartheta = 0,005$. As before, both the initial shape and the reconstructed shape (grey region) are illustrated.

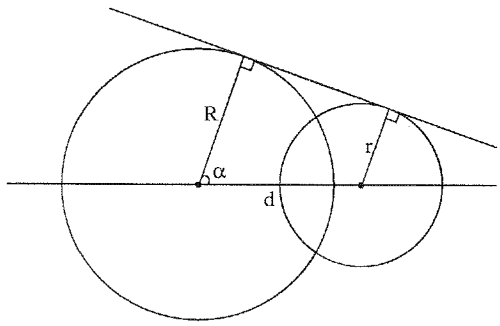

Figure 8: The area of the protrusion (dark region) can be computed in terms of the radii R and r, and of the angle α.

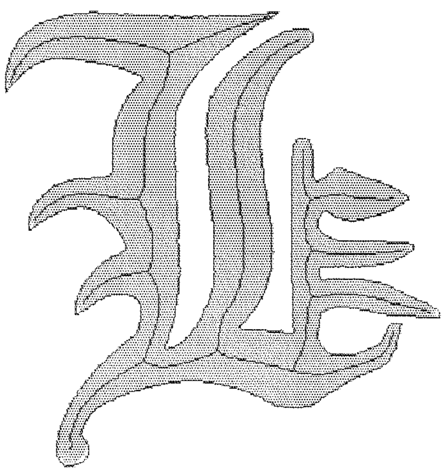

Figure 9: Effect of the area-based pruning on the discrete skeleton.

5 Conclusion

Both the discrete skeleton and the continuous skeleton suffer for the presence of a number of peripheral branches having a disregardable representative power. Removing these unwanted branches, while leaving as much as possible unmodified the remaining ones, is a crucial task, indispensable to effectively use the skeleton for pattern recognition. Pruning techniques to simplify skeleton structure without altering significantly the representative power of the skeleton have been discussed in this paper.
Pruning is useful not only to remove branches corresponding to non significant regions, but also to reduce the effect of rotation. Figure rotation has a relevant effect on the discrete skeleton, as far as peripheral branches are concerned, even if the Euclidean distance is used for its computation. Figure rotation may also influence the position and number of peripheral branches in the continuous skeleton, depending on how the sampling points are selected on the boundary.
Different criteria, based on protrusion elongation, sharpness and area, have been proposed so as to be able to deal with different problems. The suggested criteria have been implemented to prune both the discrete and the continuous skeleton. If the selected discrete skeleton is computed by using a weighted distance providing a good approximation of the Euclidean distance, the obtained results are comparable. Although the discrete and the continuous skeletons may be rather different before pruning, they consist of the same number of branches after pruning and, in both cases, are adequate to represent the initial figure.

6 Reference

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