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Mixed covering of trees and the augmentation problem with odd diameter constraints

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Abstract

In this talk, we will outline a polynomial time algorithm for solving the problem of partial covering of trees with n_1 balls of radius R_1 and n_2 balls of radius R_2 ($R_1 < R_2$) so as to maximize the total number of covered vertices. We will then show that the solutions provided by this algorithm in the particular case $R_1 = R - 1, R_2 = R$ can be used to obtain for any integer $\delta > 0$ a factor $(2 + \frac{1}{\delta})$ approximation algorithm for solving the following augmentation problem with odd diameter constraints $D = 2R+1$: given a tree T, add a minimum number of new edges such that the augmented graph has diameter $\leq D$. The previous approximation algorithm of Ishii, Yamamoto, and Nagamochi (2003) has factor 8.

Keywords: Partial Covering, Diameter, Augmentation problem, Dynamical programming, Approximation algorithms.

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This talk will be divided into two parts. In the first part, we describe a dynamic programming algorithm for solving the following problem :

Problem Partial Mixed Covering: *Given a tree $T = (V, E)$ with n vertices, the non-negative integers R_1, R_2 ($R_1 < R_2$) and n_1, n_2, locate n_1 balls of radius R_1 and n_2 balls of radius R_2 so as to maximize the total number of covered vertices.*

This problem generalizes the Maximum Coverage problem investigated by Megiddo, Zemel, and Hakimi [6], in which, given a tree T and the integers R_0 and n_0, one wish to locate n_0 balls of radius R_0 so as to maximize the total number of covered vertices. Our algorithm follows the main lines of the algorithm from [6] and works in general in the following way. Root the tree T at an arbitrary vertex u. The algorithm proceeds this tree in a upward manner, from leaves to the root, by solving larger and larger subproblems of the following type. Given the current vertex s, and the integers $0 \leq n'_1 \leq n_1, 0 \leq n'_2 \leq n_2$, the algorithm finds the *maximal* number of covered vertices of T_s in a partial covering using n'_1 balls of radius R_1 and n'_2 balls of radius R_2 located in T_s. However, the algorithm must take care of two things: (i) some ball which will be located outside T_s at some later stage and whose radius and center are yet unknown may have an impact on the covering of T_s, and (ii) we have to consider the interaction between the subtrees rooted at the neighbors of s, because some vertices of one or several such subtrees may be covered by a ball located in another subtree. To overcome these difficulties, we introduce two additional parameters r and a which take integer values in the ranges $[-1, R_2 - 1]$ and $[0, R_2]$, respectively. For fixed values of r and a, the algorithm returns the maximal number of covered vertices of T_s in a partial covering using n'_1 balls of radius R_1 and n'_2 balls of radius R_2 located in T_s (permanent balls), given that one additional (temporary) ball of radius r is located at s and that at least one of the permanent balls located in T_s covers every vertices outside T_s at distance at most a from s. This requires the solution of a resource allocation problem, which optimally distributes the balls of radius R_1 and the balls of radius R_2 among the subtrees rooted at the neighbors of s in T_s, using for this the optimal solutions of the previously solved subproblems at each of the sons of s.

Notice also that running the algorithm for Partial Mixed Covering for all feasible pairs (n'_1, n'_2), we obtain a polynomial time algorithm for the following problem:

Problem Mixed Covering: *Given a tree $T = (V, E)$ with n vertices, a function f of two non-negative integer variables, the non-negative integers R_1, R_2 $(R_1 < R_2)$ and n_1, n_2, find a covering (if it exists) of T with $n'_1 \leq n_1$ balls of radius R_1 and $n'_2 \leq n_2$ balls of radius R_2 minimizing the function $f(n'_1, n'_2)$.*

In the second part of our talk, we use this polynomial time algorithm to derive an approximation algorithm for the following augmentation problem:

Problem ADC (Augmentation under Diameter Constraints): *Given a graph $G = (V, E)$ with n vertices and a positive integer D, add a minimum number* OPT *of new edges E' such that the augmented graph $G' = (V, E \cup E')$ has diameter at most D.*

Due to its practical importance for improving the reliability of existing communication networks, the Augmentation under Diameter Constraints problem has received much attention in the literature [1,3,4,5,7]. In particular, it was shown to be NP-hard for any $D \geq 2$ and at least as difficult to approximate as SET COVER [1,5,7].

For the problem ADC on trees, Chepoi and Vaxès [1] presented a factor 2 approximation algorithm for even $D = 2R$ and Ishii, Yamamoto, and Nagamochi [4] presented a factor 8 approximation algorithm for odd $D = 2R+1$. The algorithm in [1] for $D = 2R$ computes an optimal covering of the tree T with one ball of radius R and a minimum number of balls of radius $R-1$ and then, it adds edges between the center of the radius R ball and the centers of radius $R-1$ balls in the covering. For the case $D = 2R+1$ we consider the following feasible augmentation: take a mixed covering of T with n_1 balls of radius $R-1$ and n_2 balls of radius R minimizing the function $f(n_1, n_2) = n_1 + \frac{n_2(n_2-1)}{2}$ (in this case, Mixed Covering can be solved in time $O(n^{3.5}R^2)$) and draw an edge between any pairs of centers of balls of radius R and between the centers of any balls of radius $R-1$ and the center of some ball of radius R. So, in case $D = 2R$ the added edges form a star while in case $D = 2R+1$ they constitute a clique of size n_2 plus n_1 pendant edges connected to this clique. In this talk based on the paper [2], we will outline the proof that the algorithm for odd D provides a feasible solution containing at most $(2+\frac{1}{\delta})OPT + O(\delta^5)$ added edges for any $\delta > 0$, thus asymptotically matching the approximation ratio for even D. We conjecture that this algorithm for odd D as well as the algorithm in [1] for even D are optimal up to an additive constant error term.

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