Outline

- Conway's Game of Life and its complexity
- Gandy's result on cellular automata versus our physical world
- Universality of Rule 110 in one dimension
- Current work with Sara Riva and Enrico Formenti

Conway's Game of Life

Golly `http://golly.sourceforge.net/`

Life Wiki `https://www.conwaylife.com/wiki/`

Questions on the Game of Life

Garden of eden $\rightarrow$ unknown $(6 \times 6 < \cdot \leq 8 \times 12)$

What is the smallest garden of eden configuration?

Fortress $\rightarrow$ unknown (does it exist?)

What is the smallest fortress configuration?

Death problem $\rightarrow$ undecidable

Given a finite configuration, will all cells eventually die?

watch life in life

Formal definition

Definition A cellular automaton is defined by

- a dimension d
- a finite set of states S
- a finite neighborhood $N = (n_1, n_2, \ldots, n_m)$ an m-tuple of $\mathbb{Z}^d$
- a local rule $f : S^m \to S$

A configuration is $c : \mathbb{Z}^d \to S$ and it evolves to c' as

$$\forall x \in \mathbb{Z}^d : c'(x) = f(c(x + n_1), c(x + n_2), \ldots, c(x + n_m))$$

you think GOL is a lucky example? we can discuss larger than life

Cellular automata versus our physical world

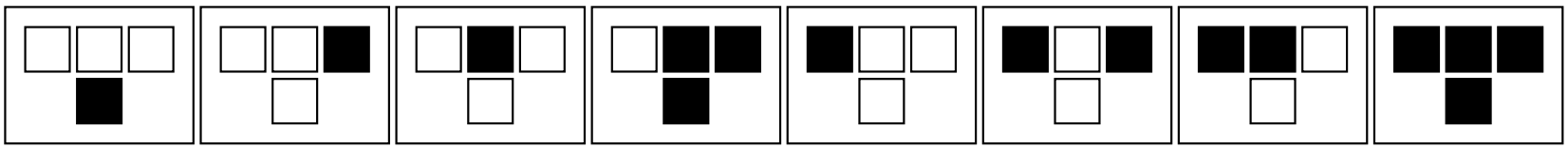
Theorem (Gandy 1980) If the following assumptions are true

1. our physical world is homogeneous in space
2. our physical world is homogeneous in time
3. velocity of propagation of information is bounded
4. density of information is bounded
5. there is quiescent state (something as "outer space")

then **we live in a cellular automaton** 

There is a quantum version of this result (Arrighi Dowek 2012)

Rule 110



Theorem (Cook 2004) Rule 110 is Turing-universal

Open Is Rule 54 Turing-universal?