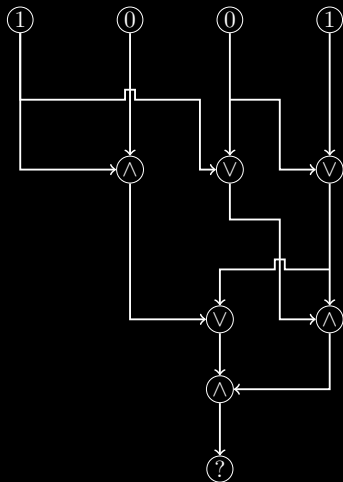


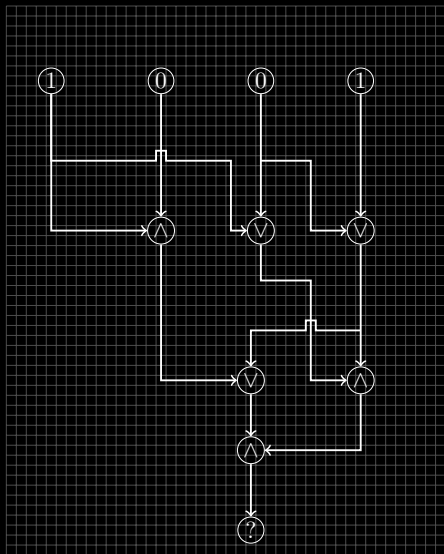
# Complexity of prediction: proving P-completeness

**Monotone Circuit Value Problem**  $\leq_{\log\text{space}}$  **Fuze circuits**



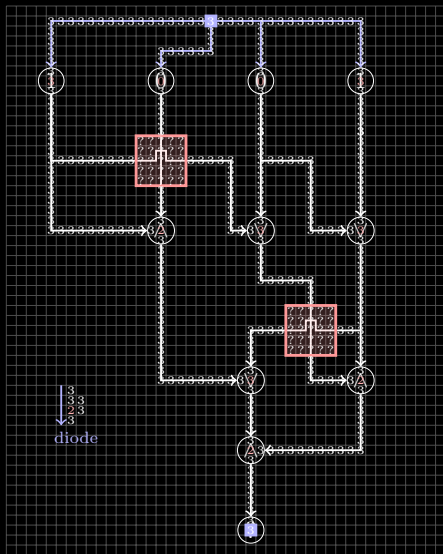
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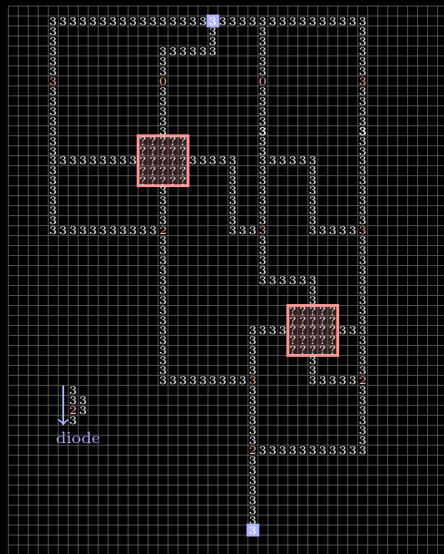
**Monotone Circuit Value Problem**  $\leq_{\log\text{space}}$  **Fuze circuits**





# Complexity of prediction: proving P-completeness

**Monotone Circuit Value Problem**  $\leq_{\log\text{space}}$  **Fuze circuits**



But **Planar Monotone CVP**  
is in **NC** [Yang, 1991].

Planar  $\iff$  Monotone

Crossover gate ?

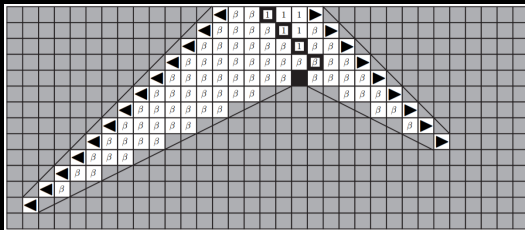
# Computing with sand

**Theorem.** Chip Firing Games on arbitrary infinite graphs (of small degree) are Turing-complete (infinite configurations).

**Theorem.** Sandpiles on  $\mathbb{Z}^d$  with dimension  $d \geq 3$  are Turing-complete (infinite ultimately periodic configurations). Even on  $\{1, 2, 3\} \times \mathbb{Z}^2$ .

**Corollary.** Undecidable problems on ultimately periodic configurations.

Sandpile Halting Problem: fuze circuits simulating a lazy CA



Goles, Margenstern: “Universality of the chip-firing game”, 1995.

Cairns: “Some halting problems for abelian sandpiles are undecidable in dimension three”, 2018.