ON PARTIALLY COMPOSED PROPERTY OF GENERALIZED LEXICOGRAPHIC PRODUCT GRAPHS

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October 10, 2017
Overview

1. Introduction
   - Basic Definitions
   - Graphical Properties
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   - Generalized Partially Composed Vertex Properties
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   - \( m_P \) and \( M_P \)
   - Applications
Introduction
Definitions and Notations

$G = (V(G), E(G))$.

$G[S]$ the subgraph of $G$ induced by $S \subseteq V(G)$.

$d(a, b, c, G)$. $G[S] = \{a, b, c\}$. $G[S]$.
Definitions and Notations

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$G \langle S \rangle$ is the subgraph of $G$ induced by $S$ in $V(G)$. 

$G \langle a; b; c \rangle = G \langle S \rangle$. 

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$G$

$G\langle S \rangle$
Definitions

Projections

For sets \( A \) and \( B \), if \( S \subseteq A \cup B \), we define

\[
1(S) = \{ a \in A : (a, b) \in S \} \quad \text{and} \quad 2(S) = \{ b \in B : (a, b) \in S \}.
\]

Isolated Vertices in \( G \langle S \rangle \).

If \( S \subseteq V \) is nonempty, we define

\[
I_G(S) = \{ v \in S : v \text{ is an isolated vertex of } G \langle S \rangle \}.
\]

\[
J_G(S) = S \setminus I_G(S).
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If $S \subseteq V$ is nonempty, we define

- $I_G(S) = \{v \in S : v$ is an isolated vertex of $G\langle S \rangle\}$.
- $J_G(S) = S - I_G(S)$. 
Lexicographic Product

Given two graphs $G$ and $H$; a lexicographic product of $G$ and $H$, denoted by $G \circ H$, is a graph with the vertex set $V(G) \times V(H)$ such that two vertices $(x_1, h_1)$ and $(y_2, h_2)$ are adjacent whenever (1) $x_1 = y_2$ and $h_1 h_2 \in E(H)$, or (2) $x_1 y_2 \in E(G)$.
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\[ \begin{array}{c}
G \\
H 
\end{array} \]
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**Diagram:**

- $G$: A graph with three vertices and two edges.
- $H$: A graph with two vertices and one edge.

The lexicographic product $G \circ H$ combines the structures of $G$ and $H$ according to the definitions provided.
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**Diagram:**

- Graph $G$ with 6 vertices and 9 edges.
- Graph $H$ with 3 vertices and 3 edges.

- The lexicographic product $G \circ H$ is shown with the combined set of vertices and edges from both $G$ and $H$.
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\[
\begin{align*}
G \circ H & \\
\text{(Diagram of } G \circ H) & \\
H & \\
G & 
\end{align*}
\]
Generalized Lexicographic Product

Given graphs $G$ and $H$, for every $x \in V(G)$, a generalized lexicographic product of $G$ and $(H_x \times x)$, denoted by $G \circ (H_x \times x)$, is a graph with the vertex set $\bigcup_{x \in V(G)} (f \times x \times g \times (H_x \times x))$ such that two vertices $(x; h) \times x$ and $(y; h) \times y$ are adjacent whenever (1) $x = y$ and $h \times h \in E(H_x \times x)$, or (2) $x \times y \in E(G)$. 
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1. $x = y$ and $h_x h_y \in E(H_x)$,
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E E e e e
e H x e e e H y e e e H z
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- $G$
- $x$
- $y$
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Graphical Properties

$\mathcal{U}$ denotes the class of all finite simple graphs. A graphical property means a nonempty isomorphism-closed subclass of $\mathcal{U}$. For graphical properties $I_1$ and $I_2$ we define $I_1 \circ I_2 = \{G \circ H : G \in I_1 \text{ and } H \in I_2\}$ when we refer $\circ$ as a lexicographic product. $I_1 \circ I_2 = \{G \circ (H \times x) : G \in I_1 \text{ and } H \times x \in I_2\}$ for all $x \in V(G)$ when we refer $\circ$ as a generalized lexicographic product.
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\[
I_1 \circ I_2 = \{G \circ (H_x) : x \in V(G) \}
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  - \( \mathcal{I}_1 \circ \mathcal{I}_2 = \{ G \circ H : G \in \mathcal{I}_1 \text{ and } H \in \mathcal{I}_2 \} \) when we refer \( \circ \) as a lexicographic product.
Graphical Properties

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  - $\mathcal{I}_1 \circ \mathcal{I}_2 = \{ G \circ (H_x)_{x \in V(G)} : G \in \mathcal{I}_1 \text{ and } H_x \in \mathcal{I}_2 \text{ for all } x \in V(G) \}$ when we refer $\circ$ as a generalized lexicographic product.
For a graph $G$, $S(G)$ denotes the class of all nonempty subsets of $V(G)$. A vertex property is a nonempty subclass of $\bigcup_{U \in \mathcal{G}} S(G)$. For a vertex property $P$ and a nonempty subset $S$ of $V(G)$, $S$ is called a $P$-set of $G$ if $S \in P$.

For a graphical property $I$, a vertex property $P$ is said to be appearing in $I$, whenever there is a $P$-set of $G$ for each $G \in I$.

Given a graphical property $I$ and a vertex property $P$ appearing in $I$; for a graph $G \in I$, $M_P(G)$ denotes the maximum cardinality of a $P$-set of $G$ and $m_P(G)$ denotes the minimum cardinality of a $P$-set of $G$. If $S$ is a $P$-set of a graph $G$ such that $|S| = M_P(G)$ or $|S| = m_P(G)$, we say that $S$ is an $M_P$-set or an $m_P$-set of $G$, respectively.
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For a graphical property $\mathcal{I}$, a vertex property $\mathcal{P}$ is said to be *appearing in $\mathcal{I}$*, whenever there is a $\mathcal{P}$-set of $G$ for each $G \in \mathcal{I}$. 

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Vertex Properties

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- Given a graphical property $\mathcal{I}$ and a vertex property $\mathcal{P}$ appearing in $\mathcal{I}$; for a graph $G \in \mathcal{I}$,
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For a vertex property $\mathcal{P}$ and a nonempty subset $S$ of $V(G)$, $S$ is called a $\mathcal{P}$-set of $G$ if $S \in \mathcal{P}$.

For a graphical property $\mathcal{I}$, a vertex property $\mathcal{P}$ is said to be appearing in $\mathcal{I}$, whenever there is a $\mathcal{P}$-set of $G$ for each $G \in \mathcal{I}$.

Given a graphical property $\mathcal{I}$ and a vertex property $\mathcal{P}$ appearing in $\mathcal{I}$; for a graph $G \in \mathcal{I}$,

- $M_{\mathcal{P}}(G)$ denotes the maximum cardinality of a $\mathcal{P}$-set of $G$
- $m_{\mathcal{P}}(G)$ denotes the minimum cardinality of a $\mathcal{P}$-set of $G$.

If $S$ is a $\mathcal{P}$-set of a graph $G$ such that $|S| = M_{\mathcal{P}}(G)$ or $|S| = m_{\mathcal{P}}(G)$, we say that $S$ is an $M_{\mathcal{P}}$-set or an $m_{\mathcal{P}}$-set of $G$, respectively.
Generalized Partially Composed Vertex Properties

Given graphical properties $I_1$ and $I_2$, a vertex property $P_1$ appearing in $I_1$, a vertex property $P_2$ appearing in $I_2$ and a vertex property $P$ appearing in $I_1 \circ I_2$; $P$ is said to be partially composed by $P_1$ and $P_2$ if it satisfies: for any $G \subseteq I_1$; $H \subseteq I_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have $S$ is a $P$-set of $G \circ H$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_2(S)$ is a $P_2$-set of $H$ for every $a \subseteq I(G(\pi_1(S)))$.

$P$ is said to be generalized partially composed by $P_1$ and $P_2$ if it satisfies: for any $G \subseteq I_1$; $H_x \subseteq I_2$ and a nonempty subset $S$ of $V(G \circ (H_x \circ H))$, we have $S$ is a $P$-set of $G \circ (H_x \circ H)$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_2(S)$ is a $P_2$-set of $H_a$ for every $a \subseteq I(G(\pi_1(S)))$. 

Sayan Panma (CMU)
Given graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, a vertex property $P$ appearing in $\mathcal{I}_1$, a vertex property $P'$ appearing in $\mathcal{I}_2$ and a vertex property $P''$ appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$; $P$ is said to be partially composed by $P_1$ and $P_2$ if it satisfies: for any $G \in \mathcal{I}_1$; $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have $S$ is a $P$-set of $G \circ H$ if and only if $1(S)$ is a $P_1$-set of $G$ and $2(a)$ is a $P_2$-set of $H$ for every $a \in I(G \circ H)$. $P$ is said to be generalized partially composed by $P_1$ and $P_2$ if it satisfies: for any $G \in \mathcal{I}_1$; $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ (H \circ V(G)))$, we have $S$ is a $P$-set of $G \circ (H \circ V(G))$ if and only if $1(S)$ is a $P_1$-set of $G$ and $2(a)$ is a $P_2$-set of $H$ for every $a \in I(G \circ H \circ V(G))$. 
Generalized Partially Composed Vertex Properties

Given graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, a vertex property $\mathcal{P}_1$ appearing in $\mathcal{I}_1$, a vertex property $\mathcal{P}_2$ appearing in $\mathcal{I}_2$ and a vertex property $\mathcal{P}$ appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$; $\mathcal{P}$ is said to be partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$ if it satisfies: for any $G \in \mathcal{I}_1$; $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have $S$ is a $\mathcal{P}$-set of $G \circ H$ if and only if $\mathcal{P}_1(S)$ is a $\mathcal{P}_1$-set of $G$ and $\mathcal{P}_2(S)$ is a $\mathcal{P}_2$-set of $H$ for every $a \in \mathcal{I}(G)(1(S))$.

$\mathcal{P}$ is said to be generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$ if it satisfies: for any $G \in \mathcal{I}_1$; $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ (H \circ a))$, we have $S$ is a $\mathcal{P}$-set of $G \circ (H \circ a)$ if and only if $\mathcal{P}_1(S)$ is a $\mathcal{P}_1$-set of $G$ and $\mathcal{P}_2(S)$ is a $\mathcal{P}_2$-set of $H \circ a$ for every $a \in \mathcal{I}(G)(1(S))$. 

Sayan Panma (CMU)
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Given graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$,
a vertex property $\mathcal{P}_1$ appearing in $\mathcal{I}_1$,
a vertex property $\mathcal{P}_2$ appearing in $\mathcal{I}_2$ and
a vertex property $\mathcal{P}$ appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$;

$\mathcal{P}$ is said to be partially composed by $\mathcal{P}_1 \circ \mathcal{P}_2$ if it satisfies:
for any $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ H)$,
we have $S$ is a $\mathcal{P}$-set of $G \circ H$ if and only if $S$ is a $\mathcal{P}_1$-set of $G$ and
$S$ is a $\mathcal{P}_2$-set of $H$ for every $a \in I(G(S))$. 

$\mathcal{P}$ is said to be generalized partially composed by $\mathcal{P}_1 \circ \mathcal{P}_2$ if it satisfies:
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Sayan Panma (CMU)
Given graphical properties $I_1$ and $I_2$, a vertex property $P_1$ appearing in $I_1$, a vertex property $P_2$ appearing in $I_2$ and a vertex property $P$ appearing in $I_1 \circ I_2$;

- $P$ is said to be partially composed by $P_1$ and $P_2$ if it satisfies: for any $G \in I_1, H \in I_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have
Generalized Partially Composed Vertex Properties

Given graphical properties $I_1$ and $I_2$, a vertex property $P_1$ appearing in $I_1$, a vertex property $P_2$ appearing in $I_2$ and a vertex property $P$ appearing in $I_1 \circ I_2$;

- $P$ is said to be *partially composed* by $P_1$ and $P_2$ if it satisfies: for any $G \in I_1$, $H \in I_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have

  - $S$ is a $P$-set of $G \circ H$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_{2a}(S)$ is a $P_2$-set of $H$ for every $a \in l_G(\pi_1(S))$. 

- $P$ is said to be *generalized partially composed* by $P_1$ and $P_2$ if it satisfies: for any $G \in I_1$, $H \in I_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have

  - $S$ is a $P$-set of $G \circ H$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_{2a}(S)$ is a $P_2$-set of $H$ for every $a \in l_G(\pi_1(S))$. 

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Generalized Partially Composed Vertex Properties

Given graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, a vertex property $\mathcal{P}_1$ appearing in $\mathcal{I}_1$, a vertex property $\mathcal{P}_2$ appearing in $\mathcal{I}_2$ and a vertex property $\mathcal{P}$ appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$;

- $\mathcal{P}$ is said to be partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$ if it satisfies: for any $G \in \mathcal{I}_1$, $H \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have
  - $S$ is a $\mathcal{P}$-set of $G \circ H$ if and only if $\pi_1(S)$ is a $\mathcal{P}_1$-set of $G$ and $\pi_2a(S)$ is a $\mathcal{P}_2$-set of $H$ for every $a \in I_G(\pi_1(S))$.

- $\mathcal{P}$ is said to be generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$ if it satisfies: for any $G \in \mathcal{I}_1$, $H_x \in \mathcal{I}_2$ and a nonempty subset $S$ of $V(G \circ (H_x)_{x \in V(G)})$, we have
Generalized Partially Composed Vertex Properties

Given graphical properties $I_1$ and $I_2$, a vertex property $P_1$ appearing in $I_1$, a vertex property $P_2$ appearing in $I_2$ and a vertex property $P$ appearing in $I_1 \circ I_2$;

- $P$ is said to be **partially composed** by $P_1$ and $P_2$ if it satisfies: for any $G \in I_1$, $H \in I_2$ and a nonempty subset $S$ of $V(G \circ H)$, we have
  
  - $S$ is a $P$-set of $G \circ H$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_{2a}(S)$ is a $P_2$-set of $H$ for every $a \in l_G(\pi_1(S))$.

- $P$ is said to be **generalized partially composed** by $P_1$ and $P_2$ if it satisfies: for any $G \in I_1$, $H_x \in I_2$ and a nonempty subset $S$ of $V(G \circ (H_x)_{x \in V(G)})$, we have
  
  - $S$ is a $P$-set of $G \circ (H_x)_{x \in V(G)}$ if and only if $\pi_1(S)$ is a $P_1$-set of $G$ and $\pi_{2a}(S)$ is a $P_2$-set of $H_a$ for every $a \in l_G(\pi_1(S))$. 

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Generalized Partially Composed Vertex Properties

P is generalized partially composed by $P_1$ and $P_2$.

$V(G \circ (H \times x)) \times V(G)$.

$S_2 P$.

$S$ is a $P$-set of $G \circ (H \times x) \times V(G)$.

$1(S)$ is a $P_1$-set of $G$.

$G(1(S))$.

$a$.

for every $a \in I_G(1(S))$.

$2 a(S)$ is a $P_2$-set of $H_a$.

$2(S)$. 

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Generalized Partially Composed Vertex Properties

\( \mathcal{P} \) is generalized partially composed by \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \).
Generalized Partially Composed Vertex Properties

\[ V(G \circ (H_x)_{x \in V(G)}) \]

\( \mathcal{P} \) is generalized partially composed by \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \)
Generalized Partially Composed Vertex Properties

\[ V(G \circ (H_x)_{x \in V(G)}) \]

\[ S \in \mathcal{P} \]

\[ \mathcal{P} \text{ is generalized partially composed by } \mathcal{P}_1 \text{ and } \mathcal{P}_2 \]

\[ S \text{ is a } \mathcal{P} \text{-set of } G \circ (H_x)_{x \in V(G)} \]
Generalized Partially Composed Vertex Properties

$V(G \circ (H_x)_{x \in V(G)})$

$S \in \mathcal{P}$

$\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$

$S$ is a $\mathcal{P}$-set of $G \circ (H_x)_{x \in V(G)}$

if and only if
Generalized Partially Composed Vertex Properties

\( V(G \circ (H_x)_{x \in V(G)}) \)

\( S \in \mathcal{P} \)

\( \mathcal{P} \) is generalized partially composed by \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \)

\( S \) is a \( \mathcal{P} \)-set of \( G \circ (H_x)_{x \in V(G)} \)

if and only if

\( \pi_1(S) \) is a \( \mathcal{P}_1 \)-set of \( G \)

\( \pi_1(S) \in \mathcal{P}_1 \)
$V(G \circ (H_x)_{x \in V(G)})$

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Generalized Partially Composed Vertex Properties

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$\pi_1(S) \in \mathcal{P}_1$

$I_G(\pi_1(S))$
Generalized Partially Composed Vertex Properties

\( V(G \circ (H_x)_{x \in V(G)}) \)

\( S \in \mathcal{P} \)

\( \mathcal{P} \) is generalized partially composed by \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \)

\( S \) is a \( \mathcal{P} \)-set of \( G \circ (H_x)_{x \in V(G)} \)

if and only if

\( \pi_1(S) \) is a \( \mathcal{P}_1 \)-set of \( G \)

for every \( a \in I_G(\pi_1(S)) \).
**Generalized Partially Composed Vertex Properties**

- $V(G \circ (H_x)_{x \in V(G)})$

  - $S \in \mathcal{P}$

  - $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$

  - $S$ is a $\mathcal{P}$-set of $G \circ (H_x)_{x \in V(G)}$

    - if and only if

    - $\pi_1(S)$ is a $\mathcal{P}_1$-set of $G$

    - $\pi_1(S) \in \mathcal{P}_1$

    - for every $a \in I_G(\pi_1(S))$.  

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Generalized Partially Composed Vertex Properties

$\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$

$S$ is a $\mathcal{P}$-set of $G \circ (H_x)_{x \in V(G)}$ if and only if

$\pi_1(S)$ is a $\mathcal{P}_1$-set of $G$

and $\pi_2a(S)$ is a $\mathcal{P}_2$-set of $H_a$

for every $a \in I_G(\pi_1(S))$. 

\begin{align*}
V(G \circ (H_x)_{x \in V(G)}) \\
S \in \mathcal{P} \\
\pi_2a(S) \in \mathcal{P}_2 \\
I_G(\pi_{\bullet a}(S)) \\
\pi_1(S) \in \mathcal{P}_1
\end{align*}
Example of Generalized Partially Composed Vertex Properties
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets

A set $S$ of vertices is said to be a \textit{dominating set}.
Example of Generalized Partially Composed Vertex Properties

**Definition of Dominating Sets**

A set $S$ of vertices is said to be a *dominating set* if every vertex in $V - S$ is adjacent to a vertex in $S$. 
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets

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Definitions
Example of Generalized Partially Composed Vertex Properties

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A set $S$ of vertices is said to be a *dominating set* if every vertex in $V - S$ is adjacent to a vertex in $S$.

Definitions

Given dominating set $S$ of a graph $G$;
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets

A set $S$ of vertices is said to be a dominating set if every vertex in $V - S$ is adjacent to a vertex in $S$.

Definitions

Given dominating set $S$ of a graph $G$;

- $S$ is total if $G[S]$ has no isolated vertices
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets

A set $S$ of vertices is said to be a *dominating set* if every vertex in $V - S$ is adjacent to a vertex in $S$.

Definitions

Given dominating set $S$ of a graph $G$;

- $S$ is *total* if $G\langle S \rangle$ has no isolated vertices
- $S$ is *independent* if $G\langle S \rangle$ has no edges.
Example of Generalized Partially Composed Vertex Properties

Definition of Dominating Sets

A set $S$ of vertices is said to be a dominating set if every vertex in $V - S$ is adjacent to a vertex in $S$.

Definitions

Given dominating set $S$ of a graph $G$;

- $S$ is total if $G[S]$ has no isolated vertices
- $S$ is independent if $G[S]$ has no edges.
- $S$ is connected if $G[S]$ is connected.
Let $I$ denote the class of all vertex-disjoint graphs. If $I_1 = I_2 = I$ and $P_1 = P_2 = \cup G_2 I_1 f S V(G)$, then $P = \cup G_2 I_1 \circ I_2 f S V(G)$ is generalized partially composed by $P_1$ and $P_2$. 

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Introduction

Example of Generalized Partially Composed Vertex Properties

Let $I$ denote the class of all vertex-disjoint graphs. If $I_1 = I_2 = I$ and $P_1 = P_2 = \cup_{\mathcal{G}_2I_1} f\ S(\mathcal{G}) : S$ is a dominating set of $\mathcal{G}$, then $P = \cup_{\mathcal{G}_2I_1} \circ I_2 f S(\mathcal{G}) : S$ is a dominating set of $\mathcal{G}$ is generalized partially composed by $P_1$ and $P_2$. 
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs.
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}^*$ and
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}^*$ and $\mathcal{P}_1 = \mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_1} \{S \subseteq V(G) : S$ is a dominating set of $G\}$,
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}^*$ and $\mathcal{P}_1 = \mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_1} \{S \subseteq V(G) : S \text{ is a dominating set of } G\}$, then $\mathcal{P} = \bigcup_{G \in \mathcal{I}_1 \circ \mathcal{I}_2} \{S \subseteq V(G) : S \text{ is a dominating set of } G\}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$. 
Example of Generalized Partially Composed Vertex Properties

Let \( I \) denote the class of all vertex-disjoint graphs. If \( I_1 = I_2 = \{ G \in \mathcal{I} : G \) has no isolated vertices \( \} \) and \( P_1 = \bigcup_{G \in I_1} G \) and \( P_2 = \bigcup_{G \in I_2} S(G) \), then \( P = \bigcup_{G \in I_1 \circ I_2} S(G) \) is generalized partially composed by \( P_1 \) and \( P_2 \).
Example of Generalized Partially Composed Vertex Properties
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Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs.
Example of Generalized Partially Composed Vertex Properties

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Example of Generalized Partially Composed Vertex Properties

Let \( \mathcal{I}^* \) denote the class of all vertex-disjoint graphs. If \( \mathcal{I}_1 = \mathcal{I}_2 = \{G \in \mathcal{I}^* : G \) has no isolated vertices \( \} \) and \( \mathcal{P}_1 = \bigcup_{G \in \mathcal{I}_1} \{S \subseteq V(G) : S \) is a total dominating set of \( G \}\) and \( \mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_2} S(G) \),
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \mathcal{I}_2 = \{ G \in \mathcal{I}^* : G \text{ has no isolated vertices} \}$ and $P_1 = \bigcup_{G \in \mathcal{I}_1} \{ S \subseteq V(G) : S \text{ is a total dominating set of } G \}$ and $P_2 = \bigcup_{G \in \mathcal{I}_2} S(G)$, then $P = \bigcup_{G \in \mathcal{I}_1 \circ \mathcal{I}_2} \{ S \subseteq V(G) : S \text{ is a total dominating set of } G \}$ is generalized partially composed by $P_1$ and $P_2$. 
Example of Generalized Partially Composed Vertex Properties
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Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs.
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs.
If $\mathcal{I}_1 = \{ G \in \mathcal{I}^* : G \text{ is connected} \}$, $\mathcal{I}_2 = \mathcal{I}^*$ and
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs.
If $\mathcal{I}_1 = \{ G \in \mathcal{I}^* : G$ is connected $\}$, $\mathcal{I}_2 = \mathcal{I}^*$ and
$\mathcal{P}_1 = \bigcup_{G \in \mathcal{I}_1} \{ S \subseteq V(G) : S$ is a connected dominating set of $G \}$ and
$\mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_2} S(G)$,
Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \{G \in \mathcal{I}^* : G$ is connected $\}$, $\mathcal{I}_2 = \mathcal{I}^*$ and 
$\mathcal{P}_1 = \bigcup_{G \in \mathcal{I}_1} \{S \subseteq V(G) : S$ is a connected dominating set of $G\}$ and 
$\mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_2} S(G)$, then $\mathcal{P} = \bigcup_{G \in \mathcal{I}_1 \circ \mathcal{I}_2} \{S \subseteq V(G) : S$ is a connected dominating set of $G\}$ is 
generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$. 
Introduction  Example of Generalized Partially Composed Vertex Properties

Example of Generalized Partially Composed Vertex Properties

Let $I$ denote the class of all vertex-disjoint graphs. If $I_1 = I_2 = I$ and $P_1 = P_2 = \bigcup_{G \in I} f(S \subseteq V(G)) : S$ is an independent dominating set of $G$ then $P = \bigcup_{G \in I} f(S \subseteq V(G)) : S$ is independent dominating set of $G$ is generalized partially composed by $P_1$ and $P_2$. 

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Example of Generalized Partially Composed Vertex
Properties

Let \( \mathcal{I}^* \) denote the class of all vertex-disjoint graphs.
Example of Generalized Partially Composed Vertex Properties

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Example of Generalized Partially Composed Vertex Properties

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Example of Generalized Partially Composed Vertex Properties

Let $\mathcal{I}^*$ denote the class of all vertex-disjoint graphs. If $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}^*$ and $\mathcal{P}_1 = \mathcal{P}_2 = \bigcup_{G \in \mathcal{I}_1} \{S \subseteq V(G) : S \text{ is an independent dominating set of } G\}$ then $\mathcal{P} = \bigcup_{G \in \mathcal{I}_1 \circ \mathcal{I}_2} \{S \subseteq V(G) : S \text{ is an independent dominating set of } G\}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$. 
Main Results
Theorem
\textbf{Main Results}  \quad m_\mathcal{P} \quad \text{and} \quad M_\mathcal{P} \\

$m_\mathcal{P}$ and $M_\mathcal{P}$

\textbf{Theorem}

For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, 

$$
\begin{align*}
m_\mathcal{P}(G \circ (H \times 2 V(G))) &= \min \left\{ j_{\mathcal{G}}(S_j) + \sum a_2 I_{\mathcal{G}}(S_j) m_\mathcal{P}(H a_j) : S_j \text{ is a } \mathcal{P}_1 \text{-set of } G \right\}, \\
M_\mathcal{P}(G \circ (H \times 2 V(G))) &= \max \left\{ \sum a_2 J_{\mathcal{G}}(S_j) V(H a_j) + \sum a_2 I_{\mathcal{G}}(S_j) M_\mathcal{P}(H a_j) : S_j \text{ is a } \mathcal{P}_1 \text{-set of } G \right\}.
\end{align*}
$$
Theorem

For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, let $\mathcal{P}_1$ be a vertex property appearing in $\mathcal{I}_1$, 

$m_{\mathcal{P}}(\mathcal{P}_1 \circ \mathcal{P}_2) = \min \{ j_{\mathcal{G}}(S) + \sum a_{\mathcal{H}}(S) : S \text{ is a } \mathcal{P}_1\text{-set of } \mathcal{G} \}$

$M_{\mathcal{P}}(\mathcal{P}_1 \circ \mathcal{P}_2) = \max \{ \sum a_{\mathcal{H}}(S) : S \text{ is a } \mathcal{P}_1\text{-set of } \mathcal{G} \}$
Main Results

$m_P$ and $M_P$

Theorem

For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, let $\mathcal{P}_1$ be a vertex property appearing in $\mathcal{I}_1$, $\mathcal{P}_2$ be a vertex property appearing in $\mathcal{I}_2$ and
Main Results

$m_P$ and $M_P$

**Theorem**

For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$,
let $P_1$ be a vertex property appearing in $\mathcal{I}_1$,
$P_2$ be a vertex property appearing in $\mathcal{I}_2$ and
$P$ a vertex property appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$
Theorem

For graphical properties $I_1$ and $I_2$, let $P_1$ be a vertex property appearing in $I_1$, $P_2$ be a vertex property appearing in $I_2$ and $P$ a vertex property appearing in $I_1 \circ I_2$ such that $P$ is generalized partially composed by $P_1$ and $P_2$. 
**$m_P$ and $M_P$**

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For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, let $\mathcal{P}_1$ be a vertex property appearing in $\mathcal{I}_1$, $\mathcal{P}_2$ be a vertex property appearing in $\mathcal{I}_2$ and $\mathcal{P}$ a vertex property appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$ such that $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$. Further, let $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$. Then
**Theorem**

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such that $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$.
Further, let $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$. Then

1. $m_\mathcal{P}(G \circ (H_x)_{x \in V(G)}) =$
   $\min \{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \}$
$m_\mathcal{P}$ and $M_\mathcal{P}$

**Theorem**

For graphical properties $\mathcal{I}_1$ and $\mathcal{I}_2$, let $\mathcal{P}_1$ be a vertex property appearing in $\mathcal{I}_1$, $\mathcal{P}_2$ be a vertex property appearing in $\mathcal{I}_2$ and $\mathcal{P}$ a vertex property appearing in $\mathcal{I}_1 \circ \mathcal{I}_2$ such that $\mathcal{P}$ is generalized partially composed by $\mathcal{P}_1$ and $\mathcal{P}_2$. Further, let $G \in \mathcal{I}_1$ and $H \in \mathcal{I}_2$. Then

1. $m_\mathcal{P}(G \circ (H_x)_{x \in V(G)}) = \min \{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \}$

2. $M_\mathcal{P}(G \circ (H_x)_{x \in V(G)}) = \max \{ \sum_{a \in J_G(S)} |V(H_a)| + \sum_{a \in I_G(S)} M_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \}.$
Main Results

Applications

Definitions

The minimum cardinality of a dominating set in a graph $G$ is called the **domination number** of $G$ and is denoted by $\gamma(G)$. The minimum cardinality of a total dominating set in $G$ is called the **total domination number** of $G$ and is denoted by $\gamma_t(G)$. The minimum cardinality of an independent dominating set in $G$ is called the **independent domination number** of $G$ and is denoted by $\gamma_i(G)$. The minimum cardinality of a connected dominating set in $G$ is called the **connected domination number** of $G$ and is denoted by $\gamma_c(G)$. 
Applications

Definitions

- The minimum cardinality of a dominating set in $G$ is called the \textit{domination number} of $G$ and is denoted by $\gamma(G)$. 
Applications

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- The minimum cardinality of a dominating set in $G$ is called the *domination number* of $G$ and is denoted by $\gamma(G)$.
- The minimum cardinality of a total dominating set in $G$ is called the *total domination number* of $G$ and is denoted by $\gamma_t(G)$. 
Applications

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Main Results

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Applications

Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \{ |J_G(S)| + \sum_{a \in I_G(S)} m_P(H_a) : S \text{ is a } P_1\text{-set of } G \} \]
Applications

Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have
Applications

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\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- Domination Number
Applications

Theorem

$$m_P(G \circ (H_x)_{x \in V(G)}) =$$
$$\min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G \right\}$$

Applications

For simple graph $G$ and $H$ we have

- **Domination Number**
  - $$\gamma(G \circ (H_x)_{x \in V(G)}) =$$
  $$\min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} \gamma(H_a) : S \text{ is a dominating set of } G \right\}.$$
Main Results

Applications

Theorem

\[ m_\mathcal{P}(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- **Domination Number**
  \[ \gamma(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} \gamma(H_a) : S \text{ is a dominating set of } G \right\}. \]

- **Total Domination Number**
Applications

Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_P(H_a) : S \text{ is a } P_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- **Domination Number**
  \[
  \gamma(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} \gamma(H_a) : S \text{ is a dominating set of } G \right\}.
  \]

- **Total Domination Number**
  \[
  \gamma_t(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} \gamma_t(H_a) : S \text{ is a total dominating set of } G \right\}.
  \]
Applications

**Theorem**

\[
m_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) = \\
\min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \right\}
\]

**Applications**

For simple graph \( G \) and \( H \) we have
Applications

Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- Independent Domination Number
Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \{|J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G\} \]

Applications

For simple graph \( G \) and \( H \) we have

- Independent Domination Number

\[ \gamma_i(G \circ (H_x)_{x \in V(G)}) = \min \left\{ \sum_{a \in I_G(S)} \gamma_i(H_a) : S \text{ is an independent dominating set of } G \right\}. \]
Applications

Theorem

\[ m_P(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{P_2}(H_a) : S \text{ is a } P_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- Independent Domination Number
  \[ \gamma_i(G \circ (H_x)_{x \in V(G)}) = \min \left\{ \sum_{a \in I_G(S)} \gamma_i(H_a) : S \text{ is an independent dominating set of } G \right\} \]

- Connected Domination Number
Applications

Theorem

\[ m_{\mathcal{P}}(G \circ (H_x)_{x \in V(G)}) = \min \left\{ |J_G(S)| + \sum_{a \in I_G(S)} m_{\mathcal{P}_2}(H_a) : S \text{ is a } \mathcal{P}_1\text{-set of } G \right\} \]

Applications

For simple graph \( G \) and \( H \) we have

- **Independent Domination Number**
  \[ \gamma_i(G \circ (H_x)_{x \in V(G)}) = \min \left\{ \sum_{a \in I_G(S)} \gamma_i(H_a) : S \text{ is an independent dominating set of } G \right\}. \]

- **Connected Domination Number**
  \[ \gamma_c(G \circ (H_x)_{x \in V(G)}) = \gamma_c(G) \text{ if } \gamma_c(G) \geq 2. \]
Main Results

Applications

THANK YOU
References

References 1


References

References 2


