

A Step-indexed Kripke Model of Hidden State via Recursive Properties on Recursively Defined Metric Spaces

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Talk outline

- 1 Motivation:
hiding state
- 2 Logic-based hiding:
capabilities, frame and anti-frame rules
- 3 Possible worlds semantics for logic-based hiding:
a refined domain equation
solution in ultrametric spaces

Hidden state

Hidden state is a key design principle used by programmers:

An object (or module, or procedure)

- maintains an **internal, mutable data structure**,
- its **lifetime spans multiple invocations**,
- its **existence is not revealed** in the object's interface description.

Hidden state

For instance, there's hidden state in a **memory manager module**:

- the module maintains a list of free'd memory chunks, and
- clients only need to know that they obtain “unused” chunks.

Or, in a procedure that uses **memoization**:

- internal use of a hash table to cache previous calls,
- clients don't depend on the hash table's existence, and how it evolves.

Why hide state?

Hiding state has several benefits for (informal) reasoning.

- 1 **simpler specification** of the object:
specification does not involve the invariant,
- 2 **simpler reasoning** about clients:
no need to thread the object's invariant through client code,
- 3 **less restricted use** of the object:
avoids the need to track aliasing in certain cases.

There should be similar advantages in **formal** reasoning.

Hiding state in a program logic

The **logic-based approach** to information hiding

- keeps **standard semantics** of the programming language

- extends program logic with **special proof rules**

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here: lambda calculus with state,
standard operational semantics $(t|h) \mapsto (t'|h')$
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The **logic-based approach** to information hiding

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here: lambda calculus with state,

standard operational semantics $(t|h) \mapsto (t'|h')$

- extends program logic with **special proof rules**

here: Charguéraud and Pottier's type and capability system,

frame and anti-frame rules [ICFP'08; LICS'08]

Charguéraud and Pottier's types and capabilities

Capabilities describe heaps:

$$C ::= \mathbf{emp} \mid \{\sigma : \tau\} \mid C_1 * C_2 \mid \dots$$

For instance, $\{\sigma_1 : \text{ref int}\} * \{\sigma_2 : \text{ref int}\}$

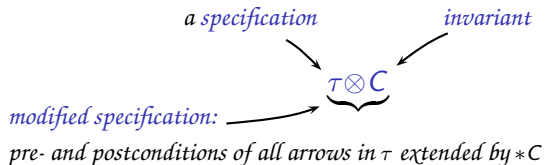
Types describe values:

$$\tau ::= \text{int} \mid [\sigma] \mid \underbrace{\tau_1 * C_1}_{\chi_1} \rightarrow \underbrace{\tau_2 * C_2}_{\chi_2} \mid \dots$$

For instance, $\text{deref} : [\sigma] * \{\sigma : \text{ref } \tau\} \rightarrow \tau * \{\sigma : \text{ref } \tau\}$

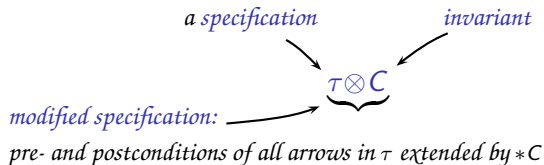
Extension of specifications by invariants

A type-theoretic connective expresses **invariant extension**:



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Formally expressed by a **type equivalence**:

- $(\chi_1 \rightarrow \chi_2) \otimes C \equiv (\chi_1 \otimes C) * C \rightarrow (\chi_2 \otimes C) * C$
- $(\tau \otimes C) \otimes C' \equiv \tau \otimes ((C \otimes C') * C')$
- $\{\sigma : \tau\} \otimes C \equiv \{\sigma : \tau \otimes C\}$

Hiding state with frame and anti-frame rules

[Shallow Frame
cf. Separation logic]

$$\frac{\Vdash t : \chi_1 \rightarrow \chi_2}{\Vdash t : \chi_1 * C \rightarrow \chi_2 * C}$$

[Deep Frame
Schwinghammer et al., CSL'09]

$$\frac{\Vdash t : \chi_1 \rightarrow \chi_2}{\Vdash t : (\chi_1 \otimes C) * C \rightarrow (\chi_2 \otimes C) * C}$$

[Anti-frame
Pottier, LICS'08]

$$\frac{\Vdash t : (\tau \otimes C) * C}{\Vdash t : \tau}$$

Explicating quantification over invariants

Intuition:

- Rules exploit **implicit** quantification over invariants.
- The semantics of arrow types makes quantification **explicit**:

$$\underbrace{\Vdash t : \chi_1 \rightarrow \chi_2}_{\text{our interpretation}} \quad \text{if} \quad \underbrace{\Vdash t : \forall C. \chi_1 \circ C \rightarrow \exists C'. \chi_2 \circ (C \circ C')}_{\text{standard interpretation}}$$

where $\cdot \circ C \stackrel{\text{def}}{=} (\cdot \otimes C) * C$.

Invariants as possible worlds

In the semantics,

- invariants C form set of worlds W ,
- capabilities and types depend on these worlds,

$$Cap \stackrel{\text{def}}{=} W \rightarrow \mathcal{P}(\text{Heap}) \quad Type \stackrel{\text{def}}{=} W \rightarrow \mathcal{P}(\text{Val}) ,$$

- invariants are arbitrary capabilities,

$$W \cong Cap .$$

Technicalities, 1

Uniform predicates $p \subseteq \mathbb{N} \times \text{Heap}$ as metric space

uniformity	$(n, h) \in p \wedge j \leq n \Rightarrow (j, h) \in p$
approximation	$p_{[n]} = \{(k, h) \in p \mid k < n\}$
distance	$d(p, q) = \inf\{2^{-n} \mid p_{[n]} = q_{[n]}\}$

Theorem (America & Rutten, 1989)

There exists a unique $W \in \text{CBUlt}$ such that

$$W \cong 1/2 \cdot W \rightarrow \text{UPred}(\text{Heap})$$

Monotonicity

Requirement:

Hidden state of non-local objects must not invalidate specifications.

Composition. Invariants can be combined:

$$\text{composition operation} \quad (c \circ c')(w) \stackrel{\text{def}}{=} (c \otimes c')(w) * c'(w)$$

$$\text{invariant extension} \quad (c \otimes c')(w) \stackrel{\text{def}}{=} c(c' \circ w)$$

Kripke monotonicity.

$w \circ w'$ is a “future world” of w : $w \leq w \circ w'$,
and capabilities need to satisfy:

$$\text{monotonicity} \quad w_1 \leq w_2 \Rightarrow c(w_1) \subseteq c(w_2)$$

Technicalities, 2

In summary, we are looking for a solution

$$\hat{W} \cong 1/2 \cdot \hat{W} \rightarrow_{mon} UPred(Heap)$$

The **definition** of the order on \hat{W} uses this isomorphism:

$$w_1 \leq w_2 \stackrel{def}{\Leftrightarrow} \exists w. w_2 = w_1 \circ w$$

$$(w_1 \circ w_2)(w) = w_1(w_2 \circ w) * w_2(w)$$

\nearrow
 \uparrow

world
capability

Consequence:

Standard existence theorems like America & Rutten's do not apply.
 Previously: tedious inverse limit construction in *CBuilt* [FOSSACS'10].

Our approach: hereditarily monotonic worlds

Theorem (Hereditarily monotonic worlds)

There exists $\hat{W} \subseteq W$ such that

$$c \in \hat{W} \Leftrightarrow \forall w_1, w_2 \in \hat{W}. c(w_1) \subseteq c(w_1 \circ w_2)$$

Proof idea:

- Consider the set Rel of non-empty closed relations $R \subseteq W$
- $Rel \in CBUlt$, when equipped with Hausdorff distance
- \hat{W} is the fixed point of contractive function $\Phi : Rel \rightarrow Rel$

Connecting the dots

Define a **step-indexed semantics** of types: **arrow types**

$$(k, \lambda x. t) \in \llbracket \chi_1 \rightarrow \chi_2 \rrbracket(w)$$

if and only if

$$\forall j < k. \forall w' \in \hat{W}.$$

$$(j, (v, h)) \in \llbracket \chi_1 \rrbracket(w \circ w') * \iota(w \circ w')(emp)$$

$$\wedge (t[x:=v]|h) \mapsto^i (t'|h') \not\mapsto$$

$$\Rightarrow \exists w'' \in \hat{W}. (j-i, (t', h')) \in \llbracket \chi_2 \rrbracket(w \circ w' \circ w'') * \iota(w \circ w' \circ w'')(emp)$$

Key ideas:

- universal and existential quantification over worlds
- using worlds as invariants
- linking uniformity and operational semantics

Summary

Frame and anti-frame rules formalize reasoning about hidden state

- specifications are “parametric” in non-local invariants
- possible-worlds model with recursive worlds
- hereditarily monotonic functions, constructed in two steps

Technically, a combination of operational and denotational ideas

- uniform predicates from step-indexing [Appel & McAllester, 2001]
- recursive metric spaces [America & Rutten, 1989]
- recursive predicates via Banach fixpoint theorem

Outlook

Study hidden state in **richer programming languages**

- continuations
- concurrency

Study Pottier's **generalized frame and anti-frame rules**

- evolving “invariants”
- parametrized recursive worlds

Thank you.